Space Market Model Space Industry Input-Output Model

Preface

This research was conducted under the auspices of the Research Institute for Computing and Information Systems by Robert F. Hodgin, Associate Professor of Economics and Finance, and Roberto Marchesini, Associate Professor of Accounting, both at the University of Houston-Clear Lake, in support of the Space Market Model research project. Overall technical direction was provided by Peter C. Bishop, Director of the Space Business Research Center, at the University of Houston-Clear Lake.

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The views and conclusions contained in this report are those of the author and should not be interpreted as representative of the official policies, either express or implied, of NASA or the United States Government.

SPACE MARKET MODEL SPACE INDUSTRY INPUT-OUTPUT MODEL

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SPACE MARKET MODEL

SPACE INDUSTRY INPUT-OUTPUT MODEL

I. INTRODUCTION

The goal of the Space Market Model (SMM) is to develop an information resource for the space industry. The SMM is intended to contain information appropriate for decision making in the space industry. As stated in the original research proposal, the objectives of the SMM are to:

- 1. Assemble information related to the development of the space business,
- 2. Construct an adequate description of the emerging space market,
- 3. Disseminate the information on the space market to forecasters and planners in government agencies and private corporations,
- 4. Provide timely analyses and forecasts of critical elements of the space market.

The first two objectives clearly indicate the need for data gathering and model building for the space industry. The last two objectives suggest how the transformed data are to be used by various agents in the economy.

This submission of the SMM report is mainly concerned with model building. The task is to develop a model of market activity capable of transforming raw data into useful information for decision makers and policy makers dealing with the space sector. The report proceeds first by describing the essential elements of an input-output (I-O) model in general. Section II discusses how the I-O structure can be modified to accommodate the space industry. Two versions are suggested. The first is a separate space industry I-O model linked generally to the rest of the economy. The second model version suggests full integration of the space sector with the national I-O model.

The final section provides insights to opportunities and barriers for both I-O model versions, summarizes the major points and argues for the fully integrated I-O model version.

II. ECONOMIC INPUT-OUTPUT MODELS

A model, in general, is a representation of actual phenomena in order to explain, predict, and control activity. In economic analysis these three functions are identified with structural analysis, forecasting, and policy evaluation, respectively.

The Leontief open, static input—output model rests on the theory of production and is designed to show the structural interdependence among sectors in an economy. The basic table of the input—output system is the transactions table. This table relates the inputs from all industries to the purchases by all other industries. The number of processing sectors depends both on the purpose of the table as well as on the availability of data. The transactions table reveals intermediate goods relationships among all industry sectors as well as sector relationships to final demand for the goods. See Table 1 for a complete schematic of a transactions table.

All input-output models rest on a series of assumptions. The most important of these are:

- Each commodity is supplied by a single industry or sector of production.
- 2. The inputs purchased by each sector are a function of the level of output of that sector.
- 3. The total effect of carrying several types of production is the sum of the separate effects.

The major quadrants of the transactions table in Table 1 deserve brief explanation. Quadrant I is labeled the processing sector. It captures the inter-industry transactions. In general, the X_{1j} term shows the sales by the ith sector, on the left, to the jth sector, at the top. Quadrant II comprises the final demand sector. Each column records the volume of sales to final purchasers by descriptive category. Quadrant III (a part of Quadrants II and IV) represents the outputs of Quadrant IV that are used as inputs to Quadrant II. In total, for the system to balance, the sum of the row totals of the payments sector must equal the sum of the column totals of the final demand sectors.

TABLE 1

INTER-INDUSTRY TRANSACTIONS TABLE

Sector	Sector Purchasing	Tobarnadia		Total					
Producing		Intermediate Goods and Services	I	Н	С	G	E	Gross Output	
P r	1	x ₁₁ x _{1j} x _{1n}	I ₁	H ₁	C ₁	G ₁	E ₁	х ₁	
o c e	2	$x_{i1}x_{ij}x_{in}$	J. (Y					
s s i	•	. (Quadrant I)	I _i	~i	1	G _i	L ₁	X _i	
n g	n	X _{n1} X _{nj} X _{nn}	^I n	H _n	C _n	G _n	En	x _n	
P	I	I ₁ I _j I _n		<u> </u>			I		
a y	H	H ₁ H _j H _n	!					Н	
m e	D	$^{\mathrm{D}}_{\mathrm{1}}$ $^{\mathrm{D}}_{\mathrm{j}}$ $^{\mathrm{D}}_{\mathrm{n}}$		v _E	D				
n t	G	G ₁ G _n (Quadrant III)							
S	М	M ₁ M _j M _n (Quadrant IV)			M				
Total Gross	Outlays	X1 Xj Xn	I	Н	С	G	E	X	

I = Inventory

G = Government

H Households

E = Exports

C = Gross Private Capital Formation

M = Imports

As described, the transactions table is a complete and detailed accounting system for an economy, typically for a year's period. Since the data are usually quite disaggregated, it reveals much more than the conventional national income accounts for an economy.

To be analytically useful, a table of technical coefficients must be constructed. Using the processing sector only, the technical coefficients are constructed in such a way that they estimate the direct purchases by each sector from every other sector per dollar of output. See the Mathematical Appendix for details. If the technical coefficients remain stable from year to year or if they can be amended on the basis of new information, it is possible to calculate the amount of direct purchases required from each industry along the left—hand side of Table 1, as a result of a change in the output of one or more of the industries listed on the top of Table 1.

Technical coefficients can change over time for three reasons:

- 1. Technical knowledge changes,
- 2. Relative prices change,
- 3. The organization of establishments changes, i.e., there is increased or decreased sub-contracting due to changes in the operational definitions of the sectors.

The table of technical coefficients, showing "first round" effects of a change in output, is also the basis for a general solution in which both direct and indirect effects on output of all industries in the processing sector can be determined.

Lastly, the true power of the Leontief structure is revealed when the model is solved generally for both direct as well as indirect effects of a change in industry sales to final demand changes. Here, estimates of both employment and income multiplier impacts can be seen.

Modification of the I-O construct, which was originally a nationally oriented schema, to a regional or even an industrial context is possible. Trade-offs exist concerning the difficulty of data collection, cost of model construction and resulting accuracy of the sectoral relationships. These issues will be discussed in the following sections.

III. AN INPUT-OUTPUT MODEL FOR THE SPACE INDUSTRY

A. Version I

The SMM is intended to capture, as much as possible, the economic behavior of the space industry. This goal is carried out by:

- 1. Defining what constitutes the space industry,
- 2. Identifying the participants in this industry,
- 3. Classifying products, inputs, revenues, costs, and overall economic activity of these agents.
- 4. Determining the major economic variables which affect the decision making process of the participants in the space industry.

In essence, the first step is to develop a structural analysis of the space sector which serves as the foundation for subsequent forecasting and policy evaluation. Two potential approaches will be discussed. The first deals with constructing a transactions matrix for the space industry and broadly linking it with the rest of the economy. The second suggests full integration of the national I-O model with the original and some newly created space related SIC coded industries and products.

TABLE 2 SAMPLE SPACE MARKET I/O STRUCTURE

Sect Purcha Sector									Final Demand						
Producing		1	2	3	4	5	6	7	I	С	G	D	E	Т	
Terrestrial Comm. N	etwork	x ₁₁			x _{1j}			X _{1n}	I ₁	c_1			E ₁	X ₁	
Launch Facilities Launch Vehicles Payload Integration		X ₁₁	:	I.	X _{ij}			X _{in}	Ii	C _i		II	Ei	x _i	
Remote Sensing Comm. Satellites Material Processing		X _{n1}			X _{nj}			Xnn	In	c _n			En	X _n	
P a	I _d	I _{d1}			^I dj			I _{dn}						I _d	
y m	A	^A 1			Aj			A _n						A	
e n	D	D ₁	IV		D _j			D _n		т	II			D	
t s	G	G ₁			Gj		•	^G n		_				G	
	M	^M 1			Мj			M n						M	
TOTAL GROSS OUTLAYS		х ₁			xj			x _n	I	С	G	D	E	Х	

I = Invent. Accumulation

C = Gross Private Capital Formation

G = Government

D = Other Domestic Sectors

E = Exports

T = Total Gross Output

M = Imports

Id = Inventory Depletion Ad = Depreciation

Although the effort involved in constructing an I-O table is extraordinary, the results are fairly easily interpreted. For illustrative purposes, Table 2 represents a possible I-O table for the SMM as it may be used to identify the main sectors which comprise it. This version assumes a processing sector for the space industry only.

The upper left-hand quadrant is labeled the processing sector and contains industries producing goods and services. For the SMM, this sector of the table includes only space and space-related industries. An industry or sector is defined as one consisting of one or more establishments producing a homogeneous product or providing a single type of service. This ideal approach is compromised, however, by practical considerations. Disclosure rules might require the aggregation of enterprises with different characteristics. SIC classification requirements would be used to assign establishments to appropriate industry sectors. In this approach, the classification of all establishments is done by using their main product classification.

In Table 2, reading across each row, the sales by the sector at the left to each of the sectors at the top are given in dollar terms. Reading down each column one observes the purchases by the sector at the top from each of the sectors listed at the left. The general term x_{ij} shows the sales by the i^{th} sector, at the left, to the j^{th} sector, at the top.

The quadrant labeled "Final Demand" constitutes the final demand sector. It shows final sales by each of the sectors at the left to the various components of final demand (C, I, G, X), the general link to the rest of the economy.

The third quadrant records the sales of primary factors to final users, while Quadrant IV is referred to generally as the payment sector. It shows the inputs from the Government, Depreciation, and Imports to each of the columns to the top of the table. Finally, Total Gross Outlays and Total Gross Output (last row and column) show that total sales to all sectors must equal total purchases by all sectors as an accounting identity.

The application of I-O analysis to the SMM demands some modifications to the basic model. First, the space sector represents one of the many producing sectors of the economy. In a national I-O table, products of the space sector would be reported under manufacturing, transportation, and so on. If all space and space-related products would be reclassified, we would derive only two vectors (row and column). The aggregation would be so severe as to make the sector meaningless. A better approach, still consistent with the assumptions of the I-O table, is to define each commodity (or commodity group) supplied by a single industry or sector of production.

It is possible, then, to separate space activities by major products or areas. Remote sensing of land areas, communication satellites, and launch vehicles are some examples. These activities would represent the cells in Quadrant I. The Final Demand sector would be represented by Inventory Accumulation, sales to Gross Private Capital Formation, Government, other Domestic sectors, and Exports (I, C, G, D, E).

The cells in Quadrant IV, the payments sector, would be constituted as:

I = Inventory Depletion

A = Depreciation

D = Domestic (other sectors)

G = Government

M = Imports

Within this framework it is possible to capture the production activities of the space industry and also to determine the effects of a change in final demand on space producing sectors. The transactions table as modified in this context, is a description of the structure of the space industry. It can be thought of as a complete and detailed accounting system for the space sectors.

Some important classification questions arise here. Within the space industry it is important to distinguish between those sub-sectors which are intermediate to other space industries and those that provide service and product to other non-space industries. As space products and services are classified according to SIC standards, these production relationships should become clear and a properly integrated transactions table can be constructed. Table 2 above is simply a first assessment as to the proper space sub-industry relationships.

Regrettably, this particular modification of an I-O model linking the space industry to the rest of the economy lacks much useful detail. Specifically, it excludes the space industry linkages to all other industries. As it stands, the construct serves as a good description of the space industry in relative isolation. How the industry is generally affected by changes in demand from the other broadly defined sectors can be assessed but no inter-industry details can be revealed.

B. Version II

Full integration of the space industry with the national I-O model is also possible. Of the six sectors identified as part of space-related activities, three of them are new activities for which no equivalent SIC classification and no equivalent sectors in the U.S. I-O model currently exist. The three are: Launch Facilities, Remote Sensing Services and Materials Processing Services.

The three remaining sectors represent activities for which SIC codes and sectors in the U.S. I-O model do exist. These sectors do not raise any serious problems for integrating the space industry with the full national model. The three sectors and related SIC categories are: Launch Vehicles (3761—Guided Missiles, Space Vehicles),

Sattelites (3663—Radio and TV Communication Equipment) and Terrestial Communications (4811—Telephone communication).

In order to deal with the three sectors not currently included, they must be precisely defined and the technical coefficients estimated. The sectoring in the U.S. model must be used to permit proper integration. This is an important rule to follow. As long as the sectors are defined along SIC bounds, the issue is not a difficult one. A set of transformation rules between the U.S. model and the space model should be delineated which would relate the two.

The SIC code defines an industry class according to both output and production characteristics with some recognition given to interindustry relationships in more heterogeneous classes. At the three and four digit SIC level, production characteristics tend to dominate even though the definitions are given by descriptions of output. Hence, similar production processes are grouped accordingly.

The three sectors which are not currently included in the SIC code or the U.S. I-O model can potentially be quantified through NASA or NASA related sources of data if no national security issues arise. The same problems of data collection and processing exist here as with field data collection. These are:

- a. Purchases are not organized or identified by industry of origin.
- b. Industry detail has been lost in compiling accounting aggregates.
- c. The amount of raw, hard copy data is very large which makes it available only through a sampling process with difficult sampling design problems attending the situation.
- d. Personnel in the organization may not be cooperative for a variety of reasons. A major reason is that they are suspicious

- of anyone wandering through the raw accounting records of their organization.
- e. Data generated by interviews is usually given at too high a level of aggregation. In order to avoid that problem, multiple interviews are required.

A possible alternate source of data may be available through the Input-Output Division of the Bureau of Economic Analysis. Either the work sheets or the individual returns of the establishments obtained from the Bureau of the Census could be quite useful. However, in order to gain access to such records, researchers must be certified as Census Officers and are subject to certain legal restrictions on disclosure and Census Bureau rules must be followed on the project.

Introducing these new sectors into the model also requires adjusting the technical coefficients of purchasing sectors in the U.S. I-O model. To relfect current technology, information would have to be obtained from the Input-Output Division of the Bureau of Economic Analysis about how they handeled these purchases in the latest U.S. model. From this information and data from the worksheets, appropriate adjustments in the purchasing sector coefficients could be made.

As to the three sectors that are already included in the U.S. I-O model, the major problem is that the SMM project may require breaking out specific activities from the U.S. sectors and setting up new sectors. If this is true, then the same data collection and processing problems exist that are discussed above.

The effect of the above activities on the technical coefficients of the current sectors would have to be removed. This task is not

impossible once technical coefficients for the space sector have been estimated. The additional information required would be the shipments of the space sector. These data combined with the shipments of the original sector could be used as weights to remove the influence of the space sector. Also, the coefficients of the purchasing sectors in the U.S. model would have to be divided between the new sectors. This would likely involve only a few sectors and would have to be based upon technical or field data. Finally, if the base year of the U.S. I-O model is different from the data collection year, adjustments should be made for changes in relative prices between the base year and the data collection year. On the more positive side, dividing up the U.S. I-O model would not be necessary if there were evidence that differences between the U.S. and space industry sectors were very small. It is not unlikely that such could be the case.

Space sector industries do represent some new establishments which are subject to rapid changes in technical knowledge until they mature. This illustrates a special case of changing technical knowledge which is related to increased experience with new processes (a learning curve phenomenon). Although perhaps difficult to handle in the early versions of a space I-O model, approximation techniques do exist.

The final result would be a new version of the U.S. I-O model showing the space industry fully integrated in enhanced detail. As a result, effects of changes in final demand could be assessed on each industry both directly and indirectly. That is, the space-related industries would be completely linked both to their own companion space sub-industries and to all other existing industries as well.

IV. SUMMARY AND CONCLUSIONS

This report has presented discussions on economic input—output models in general, the prospects for conforming the space industry to such a structure and the possibilities for how integration with the national I—O model might be accomplished. It was seen that the Leontief static, open input—output model was predicated on the theory of production. The construct permits a detailed accounting of product flows between and among industries. Additionally, the model allows an assessment of the dollar impact on the industrial structure from a change in final demand.

It was also seen that the space industry could be assessed to conform to the input-output construct in at least two ways. The first method was to build a processing sector table for all space-related industries and thereby reveal their interrelationships. These space sectors could then be linked to broad sectors of the economy. The principle drawback of this appraoch is that the desired detail of space-related industries with all other industries taken separately is obscured.

The alternative approach, suggesting full integration of the space sector with the national I-O model, provides much richer detail but requires a concomitant amount of effort. However, none of the difficulties are insurmountable. Among the most prominent issues are those concerning data collection, data processing, access to sensitive governmental data sources, the estimation of technical coefficients and the potential for restructuring some national I-O model sectors to conform to space sector definitional needs. Methods exist to manage all issues. The questions of time, cost and eventual usefulness of the results must be brought to bear before a final decision on the preferred method can be made.

Essentially, the troublesome issues fall into two areas: accuracy, especially of the technical coefficients, and base data requirements. The first issue recognizes that technical coefficients change over time, especially for those sectors where rapid technological changes take place. New information is required constantly to update the coefficients. The second issue relates that unless a transformation table exists or one can be constructed, then any linkage with the national I-O model is precluded.

Given the magnitude of the project, during Phase II of the SMM development the major tasks to be carried out would be as follows:

- 1. Final definition of the space sectors. Presently, four infrastructure sectors have been identified. They are:
 - Terrestrial Communication Network,
 - Launch Facilities,
 - Launch Vehicles.
 - Payload Integration and End-user Sectors.

Also, three end-user sectors have been identified. They are:

- Remote Sensing,
- Communication Satellites,
- Material Processing.
- 2. Idenification of all the participants of each sector (1 above.)
- 3. Development of the appropriate instruments(s) to obtain the information necessary to the construction of the transactions table. (An initial effort has been made along these lines with the "Economic Census of Satellite Remote Sensing Survey").

Task 3, above, remains the crucial point in the development of the table. If inadequate responses are received from the industry participants or official source data are unavailable, the construction of the fully integrated transactions table would not be possible.

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MATHEMATICAL APPENDIX

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Given the following balance equation

$$X_{1} = X_{11} + X_{12} + ... + X_{1n} + X_{f}$$
 (1 = 1...n)

where

X, = Total Gross Ouput

 $X_{i1} - X_{in}$ = Transactions in the endogenous sector.

 $X_f = Final Demand$

Noting that the demand for part of the output of one endogenous sector, X_1 , by another endegenous sector, X_j , is a unique function of the level of production in X_j , we have

$$X_{i,j} = a_{i,j}X_{j}$$
 (2)

Substituting equation (2) in equation (1),

$$X_{i} = a_{i1}X_{1} + a_{i2}X_{2} + ... + a_{in}X_{n} + X_{f}$$
 (i = 1...n) (3)

rewritten as

$$X_{j} = \sum_{\substack{j=1\\j=1}}^{n} a_{j} X_{j} + X_{f}$$

$$(4)$$

Technical coefficients are obtained from equation (2) by solving for a_{ij}

$$a_{jj} = \frac{x_{jj}}{x_{j}} \tag{5}$$

By dividing the entry in each column of the processing sector by the adjusted gross output for that column (total gross output minus inventory depletion) we obtain a matrix of technical coefficients, A, where

^{*} Source: Miernyk, W.H. Input-Output Analysis, N.Y.: Random House, 1965.

Technical coefficients are defined as the direct purchases by each sector from evey other sector per dollar of output. However, in order to measure the direct and indirect effects of changes in sales to final demand, that is, to measure the direct and indirect (intraindustry) requirements, we need to invert a leontief matrix which is defined as (I - A) in which I is the identity and A is the matrix of technical coefficients (6). The new matrix of coefficients showing direct and indirect effects is transposed to obtain $(I - A)_{T}^{-1}$, designated as R

$$R = \begin{bmatrix} \mathbf{r}_{11} & \cdots & \mathbf{r}_{1j} & \cdots & \mathbf{r}_{1n} \\ \mathbf{r}_{11} & \cdots & \mathbf{r}_{1j} & \cdots & \mathbf{r}_{1n} \\ \mathbf{r}_{n1} & \cdots & \mathbf{r}_{nj} & \cdots & \mathbf{r}_{nn} \end{bmatrix}$$
(7)

For any <u>new</u> final demand vector inserted into the system we use these coefficients to compute a new table of interindustry transactions as follows

$$\sum_{j=1}^{n} X_{ji} r_{ij} = X_{i}, \text{ then}$$
(8)

$$A_{ij} X_i^1 = T^1 \tag{9}$$

Equation (8) shows that we multiply each column of $(I - A)_T^{-1}$ by the new final demand associated with the corresponding row.

Each column is then summed to obtain the new total from output x_1^1 . Finally, in equation (9) each column of the table of direct input coefficients is multiplied by the new total output x_1^1 for the corresponding row. The result is the new transaction table T^1 which can be described as the new balance equation.

$$X_{i}^{1} = \sum_{i=1}^{n} a_{i,j}(X_{j}^{1}) + X_{f*}^{1} \quad (i = 1 \dots n)$$
 (10)